Introduction to Dynamic Networks
Models, Algorithms, and Analysis

Rajmohan Rajaraman, *Northeastern U.*

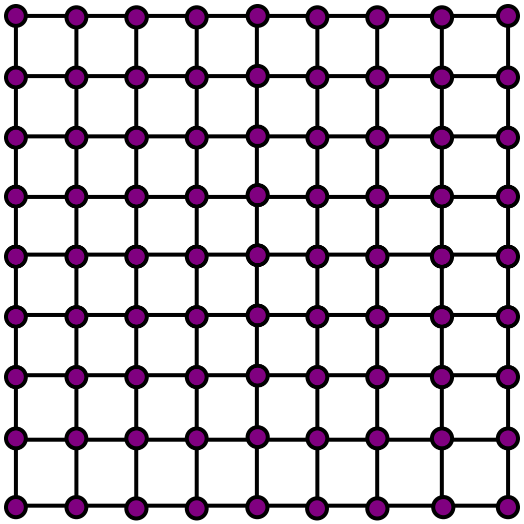
www.ccs.neu.edu/home/rraj/Talks/DynamicNetworks/DYNAMO/

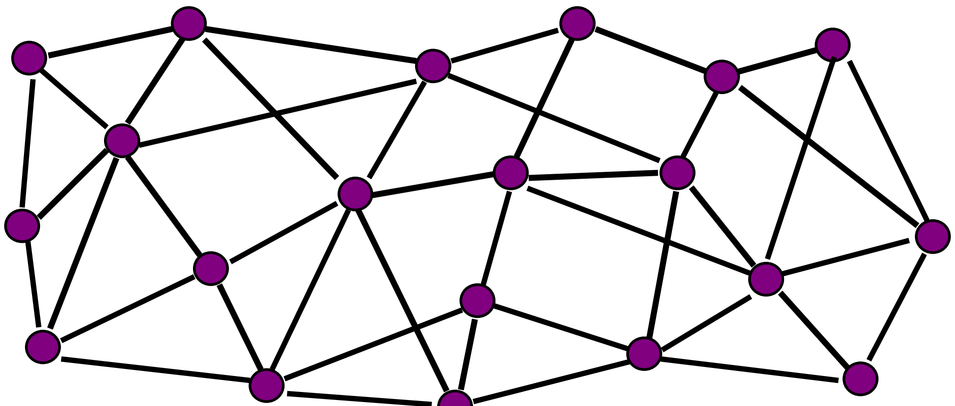
June 2006

Many Thanks to…

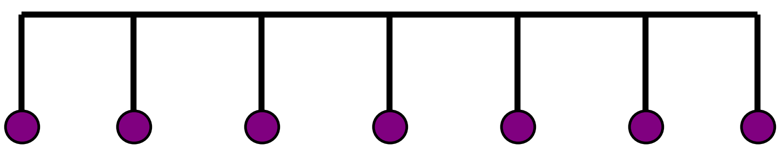
- Filipe Araujo, Pierre Fraigniaud, Luis Rodrigues, Roger Wattenhofer, and organizers of the summer school
- All the researchers whose contributions will be discussed in this tutorial

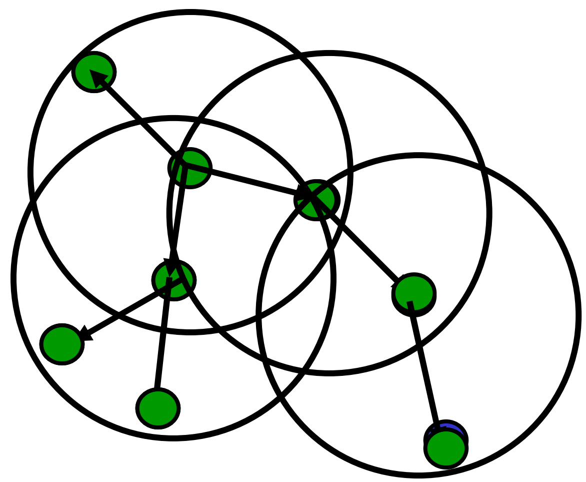
What is a Network?





General undirected or directed graph





Classification of Networks

- Synchronous:
  - Messages delivered within one time unit
  - Nodes have access to a common clock
- Asynchronous:
  - Message delays are arbitrary
  - No common clock
- Static:
  - Nodes never crash
  - Edges maintain operational status forever
- Dynamic:
  - Nodes may come and go
  - Edges may crash and recover

Dynamic Networks: What?

- Network dynamics:
  - The network topology changes over times
  - Nodes and/or edges may come and go
  - Captures faults and reliability issues
- Input dynamics:
  - Load on network changes over time
  - Packets to be routed come and go
  - Objects in an application are added and deleted

Dynamic Networks: How?

- Duration:
  - Transient: The dynamics occur for a short period, after which the system is static for an extended time period
  - Continuous: Changes are constantly occurring and the system has to constantly adapt to them
- Control:
  - Adversarial
  - Stochastic
  - Game-theoretic

Dynamic Networks are Everywhere

- Internet
  - The network, traffic, applications are all dynamically changing
- Local-area networks
  - Users, and hence traffic, are dynamic
- Mobile ad hoc wireless networks
  - Moving nodes
  - Changing environmental conditions
- Communication networks, social networks, Web, transportation networks, other infrastructure

Adversarial Models

- Dynamics are controlled by an adversary
  - Adversary decides when and where changes occur
  - Edge crashes and recoveries, node arrivals and departures
  - Packet arrival rates, sources, and destinations
- For meaningful analysis, need to constrain adversary
  - Maintain some level of connectivity
  - Keep packet arrivals below a certain rate

Stochastic Models

- Dynamics are described by a probabilistic process
  - Neighbors of new nodes randomly selected
  - Edge failure/recovery events drawn from some probability distribution
  - Packet arrivals and lengths drawn from some probability distribution
- Process parameters are constrained
  - Mean rate of packet arrivals and service time distribution moments
  - Maintain some level of connectivity in network

Game-Theoretic Models

- Implicit assumptions in previous two models:
  - All network nodes are under one administration
  - Dynamics through external influence
- Here, each node is a potentially independent agent
  - Own utility function, and rationally behaved
  - Responds to actions of other agents
  - Dynamics through their interactions
- Notion of stability:
  - Nash equilibrium

Design & Analysis Considerations

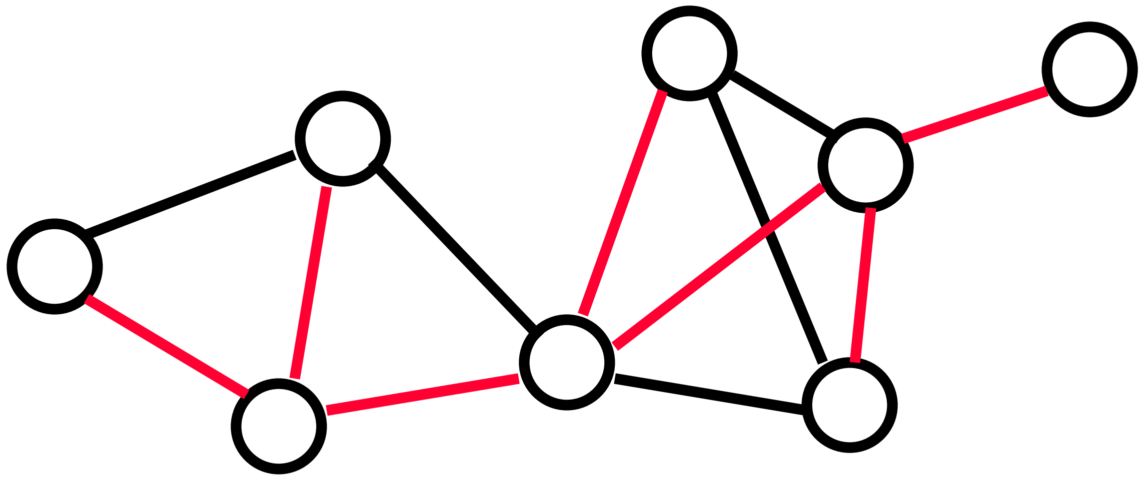
- Distributed computing:
  - For static networks, can do pre-processing
  - For dynamic networks (even with transient dynamics), need distributed algorithms
- Stability:
  - Transient dynamics: Self-stabilization
  - Continuous dynamics: Resources bounded at all times
  - Game-theoretic: Nash equilibrium
- Convergence time
- Properties of stable states:
  - How much resource is consumed?
  - How well is the network connected?
  - How far is equilibrium from socially optimal?

Five Illustrative Problem Domains

- Spanning trees
  - Transient dynamics, self-stabilization
- Load balancing
  - Continuous dynamics, adversarial input
- Packet routing
  - Transient & continuous dynamics, adversarial
- Queuing systems
  - Adversarial input
- Network evolution
  - Stochastic & game-theoretic

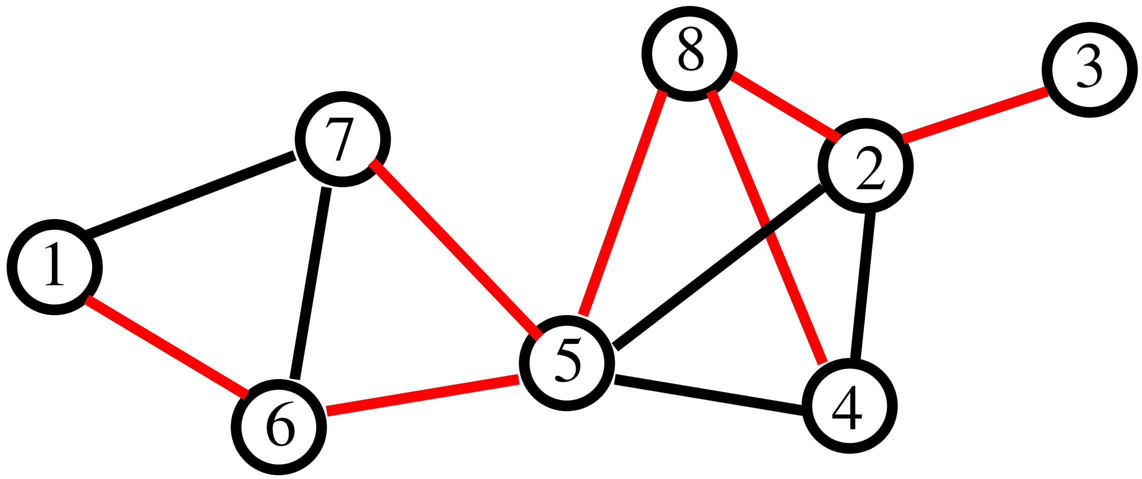
Spanning Trees

Spanning Trees



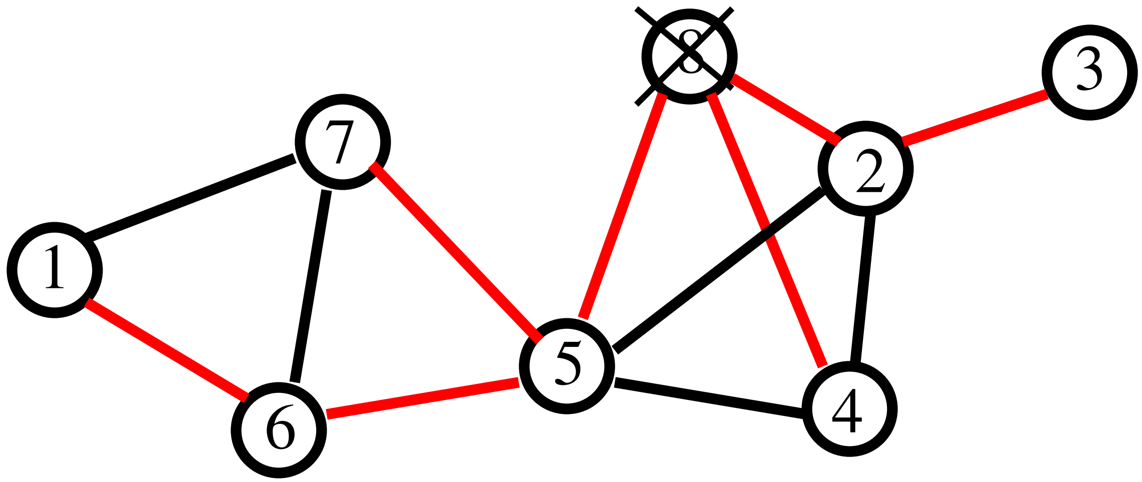
- One of the most fundamental network structures
- Often the basis for several distributed system operations including leader election, clustering, routing, and multicast
- Variants: any tree, BFS, DFS, minimum spanning trees

Spanning Tree in a Static Network



- Assumption: Every node has a unique identifier
- The largest id node will become the root
- Each node v maintains distance d(v) and next-hop h(v) to largest id node r(v) it is aware of:
  - Node v propagates (d(v),r(v)) to neighbors
  - If message (d,r) from u with r > r(v), then store (d+1,r,u)
  - If message (d,r) from p(v), then store (d+1,r,p(v))

Spanning Tree in a Dynamic Network



- Suppose node 8 crashes
- Nodes 2, 4, and 5 detect the crash
- Each separately discards its own triple, but believes it can reach 8 through one of the other two nodes
  - Can result in an infinite loop
- How do we design a self-stabilizing algorithm?

Exercise

- Consider the following spanning tree algorithm in a synchronous network
- Each node v maintains distance d(v) and next-hop h(v) to largest id node r(v) it is aware of
- In each step, node v propagates (d(v),r(v)) to neighbors
- On receipt of a message:
  - If message (d,r) from u with r > r(v), then store (d+1,r,u)
  - If message (d,r) from p(v), then store (d+1,r,p(v))
- Show that there exists a scenario in which a node fails, after which the algorithm never stabilizes

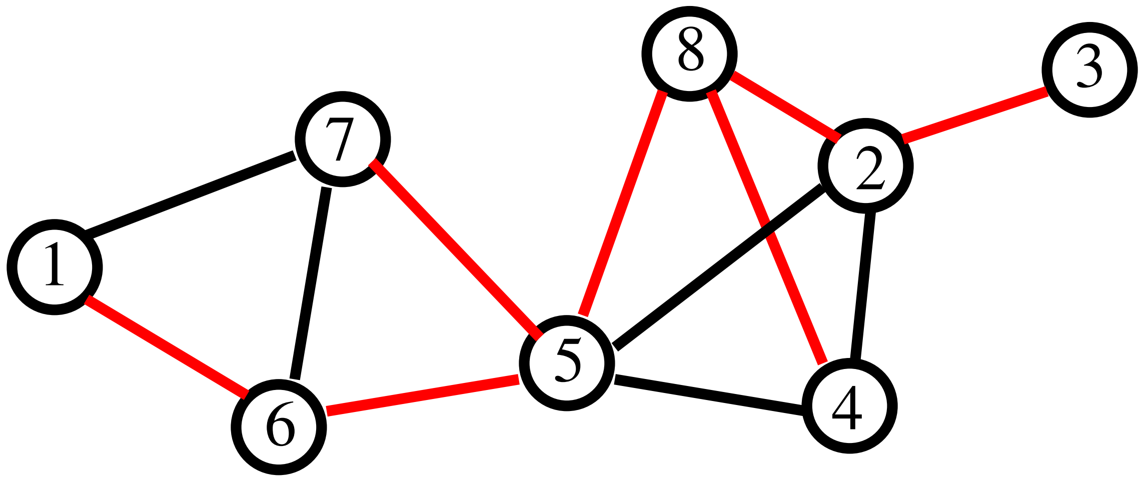
Self-Stabilization

- Introduced by Dijkstra [Dij74]
  - Motivated by fault-tolerance issues [Sch93]
  - Hundreds of studies since early 90s
- A system S is self-stabilizing with respect to predicate P
  - Once P is established, P remains true under no dynamics
  - From an arbitrary state, S reaches a state satisfying P within finite number of steps
- Applies to transient dynamics
- Super-stabilization notion introduced for continuous dynamics [DH97]

Self-Stabilizing ST Algorithms

- Dozens of self-stabilizing algorithms for finding spanning trees under various models [Gär03]
  - Uniform vs non-uniform networks
  - Fixed root vs non-fixed root
  - Known bound on the number of nodes
  - Network remains connected
- Basic idea:
  - Some variant of distance vector approach to build a BFS
  - Symmetry-breaking
    - Use distinguished root or distinct ids
  - Cycle-breaking
    - Use known upper bound on number of nodes
    - Local detection paradigm

Self-Stabilizing Spanning Tree



- Suppose upper bound N known on number of nodes [AG90]
- Each node v maintains distance d(v) and parent h(v) to largest id node r(v) it is aware of:
  - Node v propagates (d(v),r(v)) to neighbors
  - If message (d,r) from u with r > r(v), then store (d+1,r,u)
  - If message (d,r) from p(v), then store (d+1,r,p(v))
- If d(v) exceeds N, then store (0,v,v): breaks cycles

Self-Stabilizing Spanning Tree

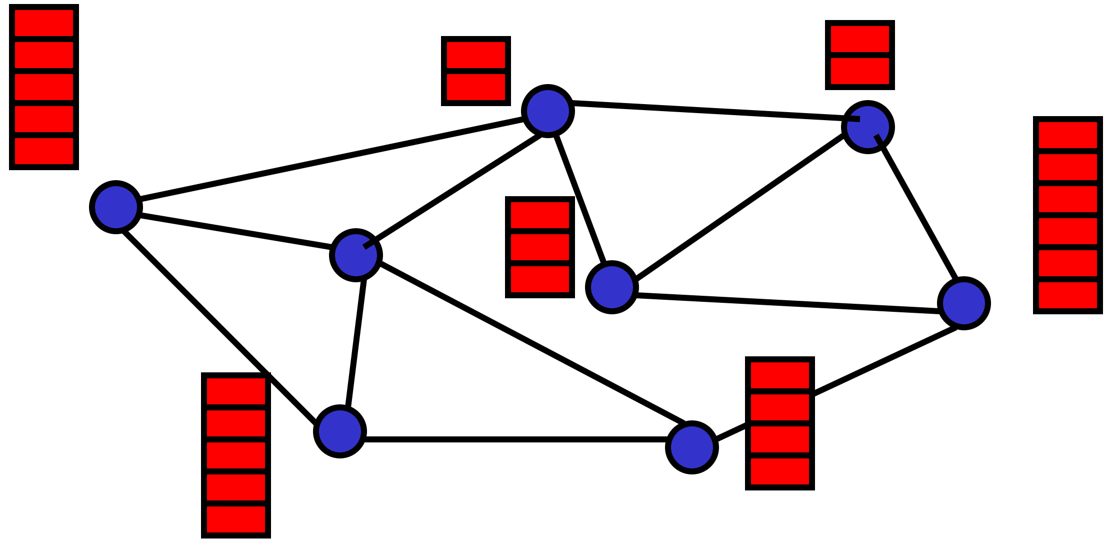
- Suppose upper bound N not known [AKY90]
- Maintain triple (d(v),r(v),p(v)) as before
  - If v > r(u) of all of its neighbors, then store (0,v,v)
  - If message (d,r) received from u with r > r(v), then v “joins” this tree
    - Sends a join request to the root r
    - On receiving a grant, v stores (d+1,r,u)
  - Other local consistency checks to ensure that cycles and fake root identifiers are eventually detected and removed

Spanning Trees: Summary

- Model:
  - Transient adversarial network dynamics
- Algorithmic techniques:
  - Symmetry-breaking through ids and/or a distinguished root
  - Cycle-breaking through sequence numbers or local detection
- Analysis techniques:
  - Self-stabilization paradigm
- Other network structures:
  - Hierarchical clustering
  - Spanners (related to metric embeddings)

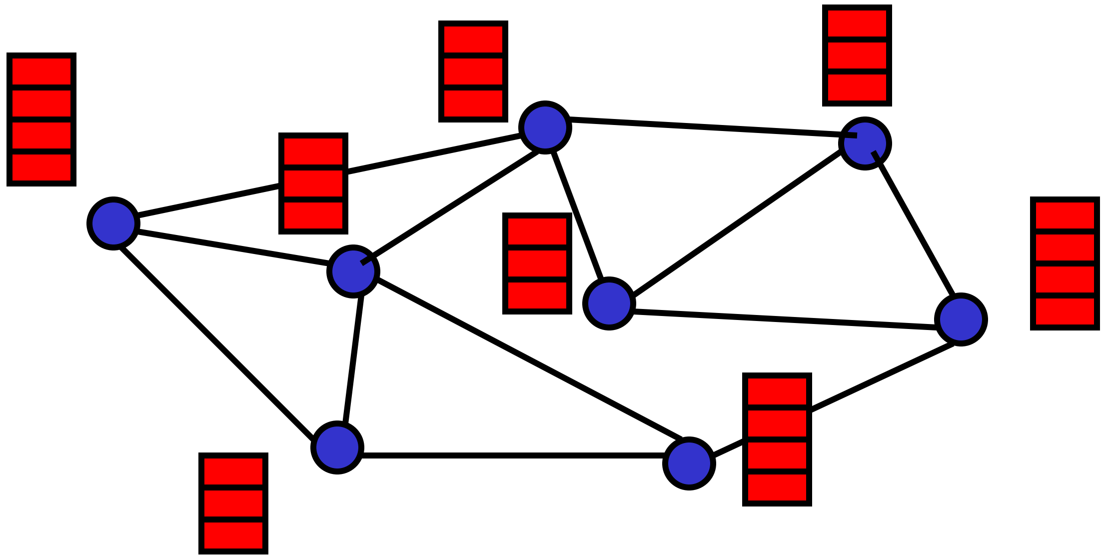
Load Balancing

Load Balancing



- Each node v has w(v) tokens
- Goal: To balance the tokens among the nodes
- Imbalance: $\max_{u,v} |w(u) - w_{avg}|$
- In each step, each node can send at most one token to each of its neighbors

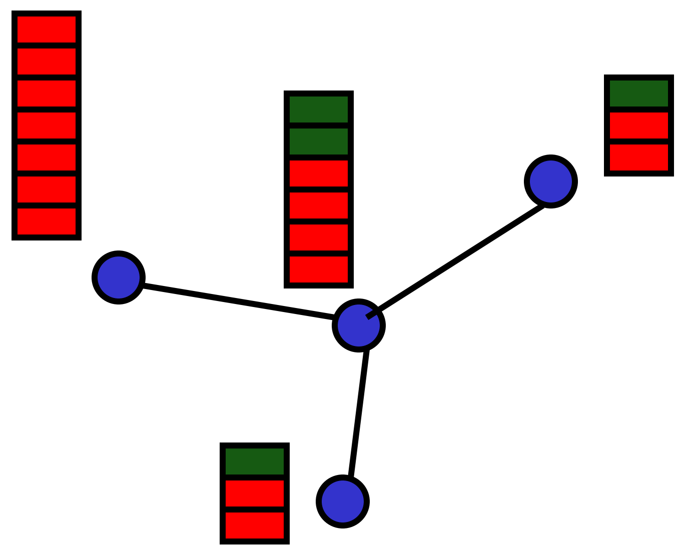
Load Balancing



- In a truly balanced configuration, we have $|w(u) - w(v)| \leq 1$
- Our goal is to achieve fast approximate balancing
- Preprocessing step in a parallel computation
- Related to routing and counting networks [PU89, AHS91]

Local Balancing

- Each node compares its number of tokens with its neighbors
- In each step, for each edge (u,v):
  - If $w(u) > w(v) + 2d$, then u sends a token to v
  - Here, d is maximum degree of the network
- Purely local operation



Convergence to Stable State

- How long does it take local balancing to converge?
- What does it mean to converge?
  - Imbalance is “constant” and remains so
- What do we mean by “how long”?
  - The number of time steps it takes to achieve the above imbalance
  - Clearly depends on the topology of the network and the imbalance of the original token distribution

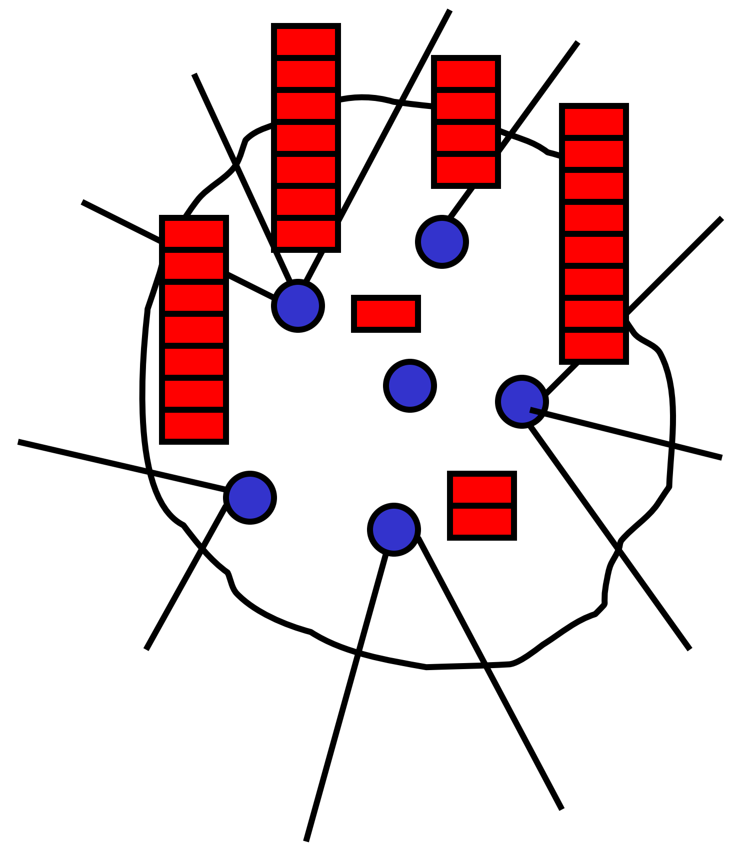
Expansion of a Network

- Edge expansion α:
  - Minimum, over all sets S of size $\leq n/2$, of the term

    $$|E(S)|/|S|$$

- Lower bound on convergence time:

  $$(w(S) - |S| \cdot w_{avg})/E(S)$$
  $$= (w(S)/|S| - w_{avg})/\ \alpha$$



Expansion = 12/6 = 2
w_{avg} = 3
Lower bound = (29 - 18)/12

Properties of Local Balancing

- For any network G with expansion α, any token distribution with imbalance Δ converges to a distribution with imbalance $O(d \cdot \log(n)/ \alpha)$ in $O(\Delta/ \alpha)$ steps [AAMR93, GLM+99]
- Analysis technique:
  - Associate a potential with every node v, which is a function of the w(v)
    - Example: $(w(v) - avg)^2$, $c^{w(v)-avg}$
    - Potential of balanced configuration is small
  - Argue that in every step, the potential decreases by a desired amount (or fraction)
  - Potential decrease rate yields the convergence time
- There exist distributions with imbalance Δ that would take $\Omega(\Delta/ \alpha)$ steps

Exercise

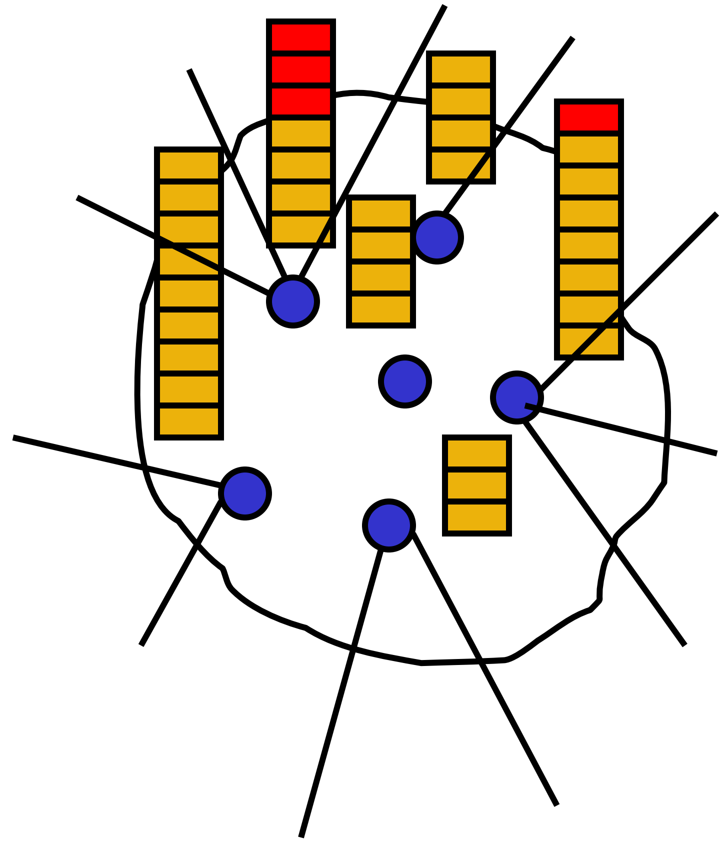
- For any graph G with edge expansion α, show that there is an initial distribution with imbalance Δ such that the time taken to reduce the imbalance by even half is $\Omega(\Delta/ \alpha)$ steps

Local Balancing in Dynamic Networks

- The "purely local" nature of the algorithm useful for dynamic networks
- Challenge:
  - May not "know" the correct load on neighbors since links are going up and down
- Key ideas:
  - Maintain an estimate of the neighbors' load, and update it whenever the link is live
  - Be more conservative in sending tokens
- Result:
  - Essentially same as for static networks, with a slightly higher final imbalance, under the assumption that the the set of live edges form a network with edge expansion α at each step

Adversarial Load Balancing

- Dynamic load [MR02]
  - Adversary inserts and/or deletes tokens
- In each step:
  - Balancing
  - Token insertion/deletion
- For any set S, let $d_t(S)$ be the change in number of tokens at step t
- Adversary is constrained in how much imbalance can be increased in a step
- Local balancing is stable against rate 1 adversaries [AKK02]



$$d_t(S) - (\mathrm{avg}_{t+1} - \mathrm{avg}_t)|S| \le r \cdot e(S)$$

Stochastic Adversarial Input

- Studied under a different model [AKU05]
  - Any number of tokens can be exchanged per step, with one neighbor
- Local balancing in this model [GM96]
  - Select a random matching
  - Perform balancing across the edges in matching
- Load consumed by nodes
  - One token per step
- Load placed by adversary under statistical constraints
  - Expected injected load within window of w steps is at most rnw
  - The pth moment of total injected load is bounded, $p > 2$
- Local balancing is stable if $r < 1$

Load Balancing: Summary

- Algorithmic technique:
  - Local balancing
- Design technique:
  - Obtain a purely distributed solution for static network, emphasizing local operations
  - Extend it to dynamic networks by maintaining estimates
- Analysis technique:
  - Potential function method
  - Martingales

Packet Routing

The Packet Routing Problem

- Given a network and a set of packets with source-destination pairs
  - Path selection: Select paths between sources and respective destinations
  - Packet forwarding: Forward the packets to the destinations along selected paths
- Dynamics:
  - Network: edges and their capacities
  - Input: Packet arrival rates and locations
- Interconnection networks [Lei91], Internet [Hui95], local-area networks, ad hoc networks [Per00]

Packet Routing: Performance

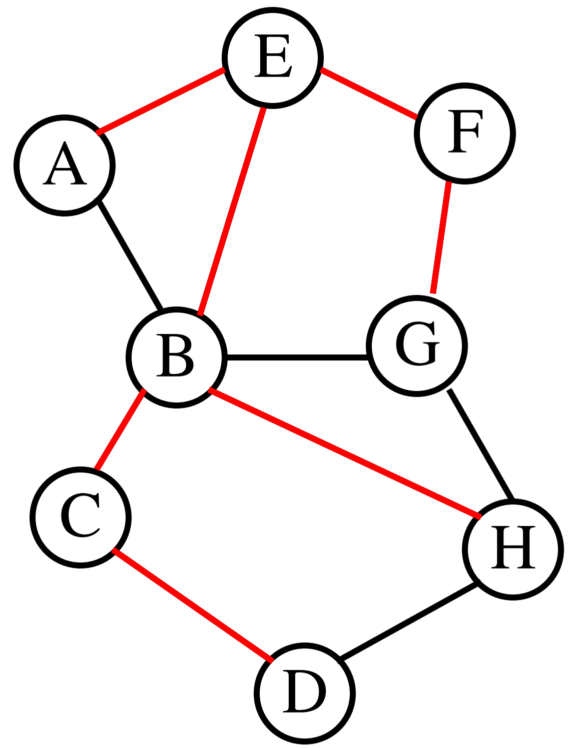
- Static packet set:
  - Congestion of selected paths: Number of paths that intersect at an edge/node
  - Dilation: Length of longest path
- Dynamic packet set:
  - Throughput: Rate at which packets can be delivered to their destination
  - Delay: Average time difference between packet release at source and its arrival at destination
- Dynamic network:
  - Communication overhead due to a topology change
  - In highly dynamic networks, eventual delivery?
- Compact routing:
  - Sizes of routing tables

Routing Algorithms Classification

- Global:
  - All nodes have complete topology information
- Decentralized:
  - Nodes know information about neighboring nodes and links
- Static:
  - Routes change rarely over time
- Dynamic:
  - Topology changes frequently requiring dynamic route updates
- Proactive:
  - Nodes constantly react to topology changes always maintaining routes of desired quality
- Reactive:
  - Nodes select routes on demand

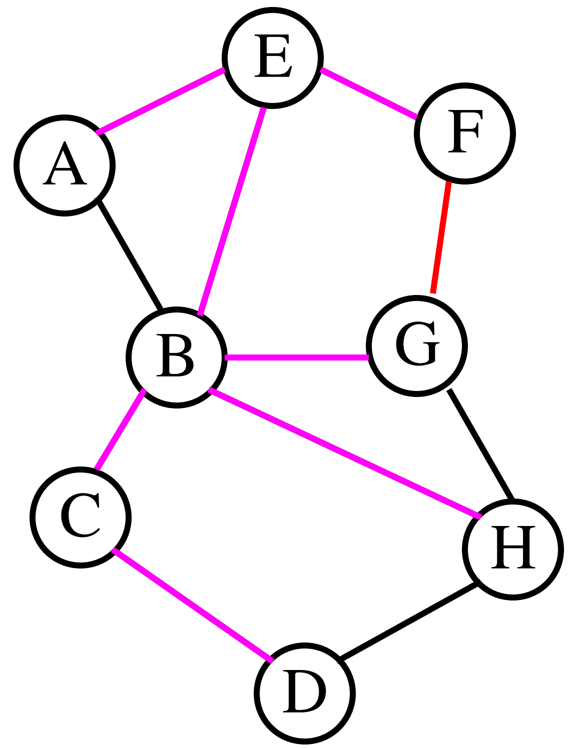
Link State Routing

- Each node periodically broadcasts state of its links to the network
- Each node has current state of the network
- Computes shortest paths to every node
  - Dijkstra's algorithm
- Stores next hop for each destination



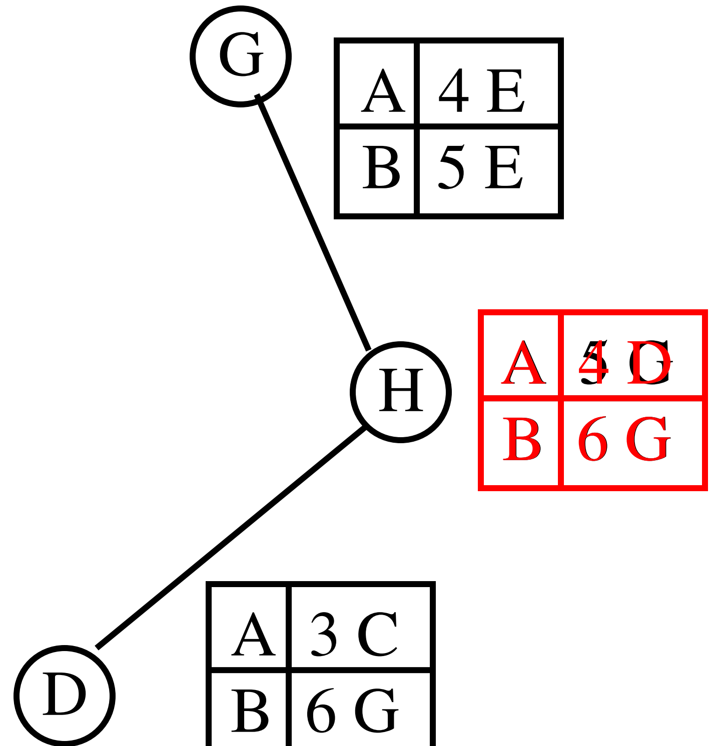
Link State Routing, contd

- When link state changes, the broadcasts propagate change to entire network
- Each node recomputes shortest paths
- High communication complexity
- Not effective for highly dynamic networks
- Used in intra-domain routing
  - OSPF



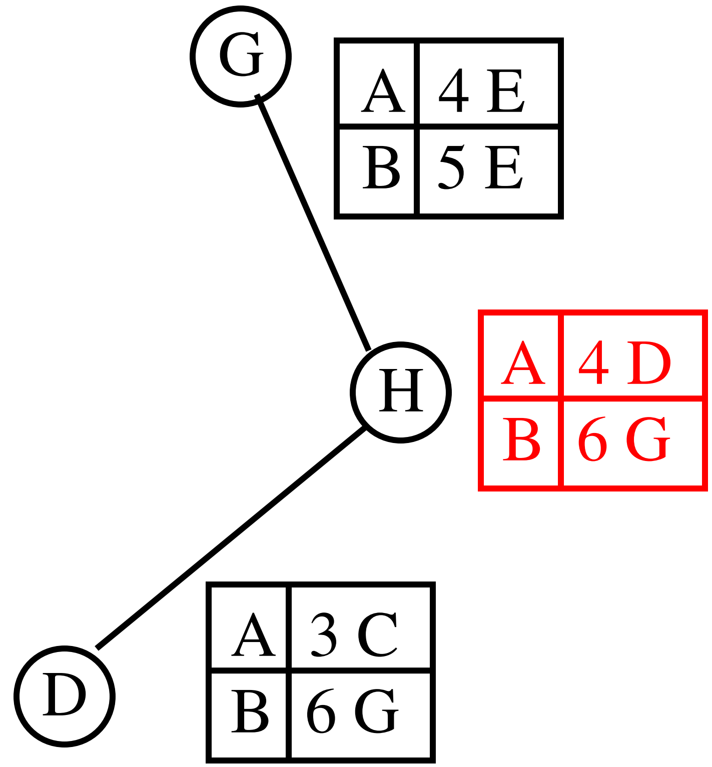
Distance Vector Routing

- Distributed version of Bellman-Ford’s algorithm
- Each node maintains a distance vector
  - Exchanges with neighbors
  - Maintains shortest path distance and next hop
- Basic version not self-stabilizing
  - Use bound on number of nodes or path length
  - Poisoned reverse



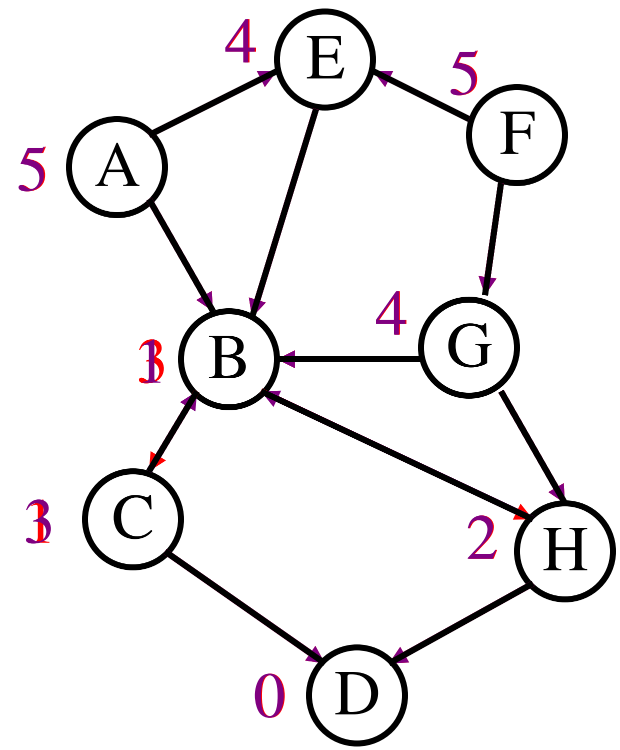
Distance Vector Routing

- Basis for two routing protocols for mobile ad hoc wireless networks
- DSDV: proactive, attempts to maintain routes
- AODV: reactive, computes routes on-demand using distance vectors [PBR99]



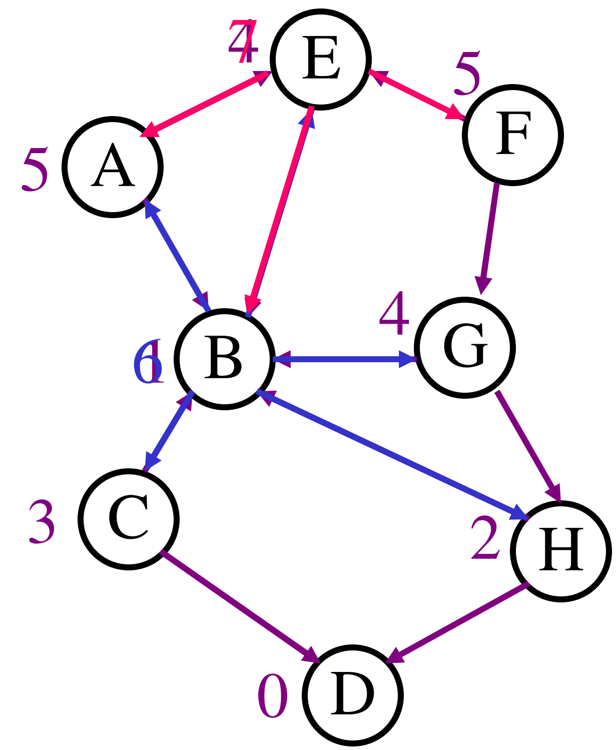
Link Reversal Routing

- Aimed at dynamic networks in which finding a single path is a challenge [GB81]
- Focus on a destination D
- Idea: Impose direction on links so that all paths lead to D
- Each node has a height
  - Height of D = 0
  - Links are directed from high to low
- D is a sink
- By definition, we have a directed cyclic graph



Setting Node Heights

- If destination D is the only sink, then all directed paths lead to D
- If another node is a sink, then it reverses all links:
  - Set its height to 1 more than the max neighbor height
- Repeat until D is only sink
- A potential function argument shows that this procedure is self-stabilizing



Exercise

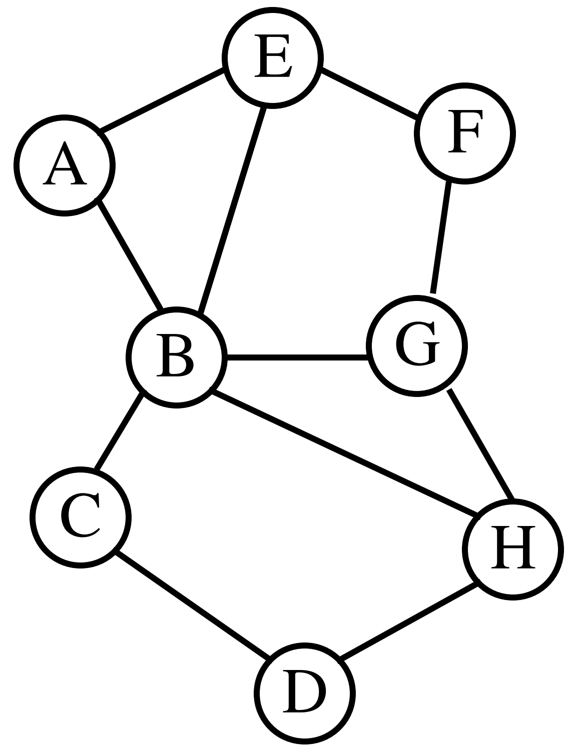
- For tree networks, show that the link reversal algorithm self-stabilizes from an arbitrary state

Issues with Link Reversal

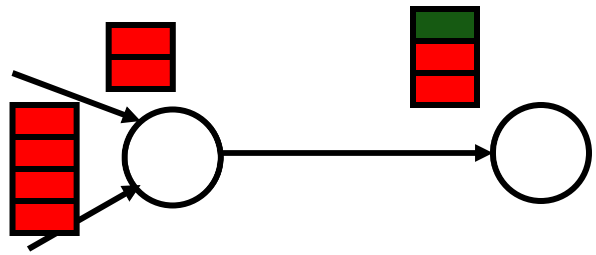
- A local disruption could cause global change in the network
  - The scheme we studied is referred to as full link reversal
  - Partial link reversal
- When the network is partitioned, the component without sink has continual reversals
  - Proposed protocol for ad hoc networks (TORA) attempts to avoid these [PC97]
- Need to maintain orientations of each edge for each destination
- Proactive: May incur significant overhead for highly dynamic networks

Routing in Highly Dynamic Networks

- Highly dynamic network:
  - The network may not even be connected at any point of time
- Problem: Want to route a message from source to sink with small overhead
- Challenges:
  - Cannot maintain any paths
  - May not even be able to find paths on demand
  - May still be possible to route!



End-to-End Communication



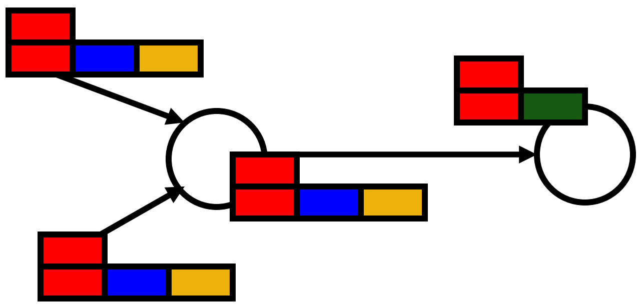
- Consider basic case of one source-destination pair
- Need redundancy since packet sent in wrong direction may get stuck in disconnected portion!
- Slide protocol (local balancing) [AMS89, AGR92]
  - Each node has an ordered queue of at most n slots for each incoming link (same for source)
  - Packet moved from slot i at node v to slot j at the (v,u)-queue of node u only if $j < i$
  - All packets absorbed at destination
  - Total number of packets in system at most $C = O(nm)$

End-to-End Communication

- End-to-end communication using slide
- For each data item:
  - Sender sends 2C+1 copies of item (new token added only if queue is not full)
  - Receiver waits for 2C+1 copies and outputs majority
- Safety: The receiver output is always prefix of sender input
- Liveness: If the sender and the receiver are eventually connected:
  - The sender will eventially input a new data item
  - The receiver eventually outputs the data item
- Strong guarantees considering weak connectivity
- Overhead can be reduced using coding e.g. [Rab89]

Routing Through Local Balancing

- Multi-commodity flow [AL94]
- Queue for each flow's packets at head and tail of each edge
- In each step:
  - New packets arrive at sources
  - Packet(s) transmitted along each edge using local balancing
  - Packets absorbed at destinations
  - Queues balanced at each node
- Local balancing through potentials
  - Packets sent along edge to maximize potential drop, subject to capacity
- Queues balanced at each node by simply distributing packets evenly



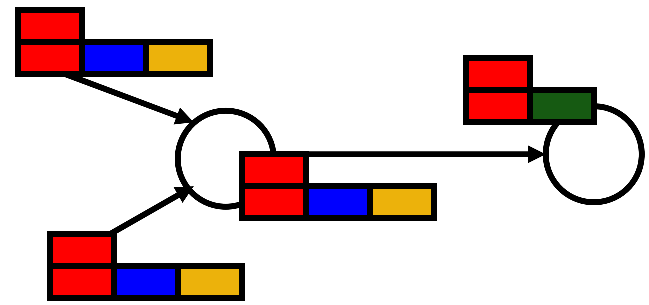
$\varphi_k(q) = \exp(\varepsilon q/(8Ld_k))$

L = longest path length

d_k = demand for flow k

Routing Through Local Balancing

- Edge capacities can be dynamically and adversarially changing
- If there exists a feasible flow that can route d_k flow for all k:
  - This routing algorithm will route (1-eps) d_k for all k
- Crux of the argument:
  - Destination is a sink and the source is constantly injecting new flow
  - Gradient in the direction of the sink
  - As long as feasible flow paths exist, there are paths with potential drop
- Follow-up work has looked at packet delays and multicast problems [ABBS01, JRS03]



$\varphi_k(q) = \exp(\varepsilon q/(8Ld_k))$

L = longest path length

d_k = demand for flow k

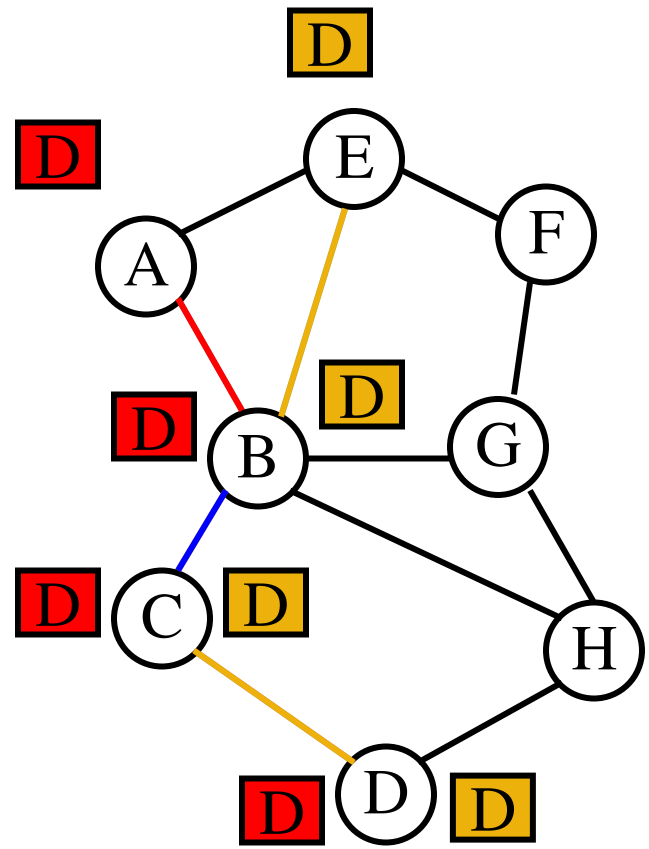
Packet Routing: Summary

- Models:
  - Transient and continuous dynamics
  - Adversarial
- Algorithmic techniques:
  - Distance vector
  - Link reversal
  - Local balancing
- Analysis techniques:
  - Potential function

Queuing Systems

Packet Routing: Queuing

- We now consider the second aspect of routing: queuing
- Edges have finite capacity
- When multiple packets need to use an edge, they get queued in a buffer
- Packets forwarded or dropped according to some order



Packet Queuing Problems

- In what order should the packets be forwarded?
  - First in first out (FIFO or FCFS)
  - Farthest to go (FTG), nearest to go (NTG)
  - Longest in system (LIS), shortest in system (SIS)
- Which packets to drop?
  - Tail drop
  - Random early detection (RED)
- Major considerations:
  - Buffer sizes
  - Packet delays
  - Throughput
- Our focus: forwarding

Dynamic Packet Arrival

- Dynamic packet arrivals in static networks
  - Packet arrivals: when, where, and how?
  - Service times: how long to process?
- Stochastic model:
  - Packet arrival is a stochastic process
  - Probability distribution on service time
  - Sources, destinations, and paths implicitly constrained by certain load conditions
- Adversarial model:
  - Deterministic: Adversary decides packet arrivals, sources, destinations, paths, subject to deterministic load constraints
  - Stochastic: Load constraints are stochastic

(Stochastic) Queuing Theory

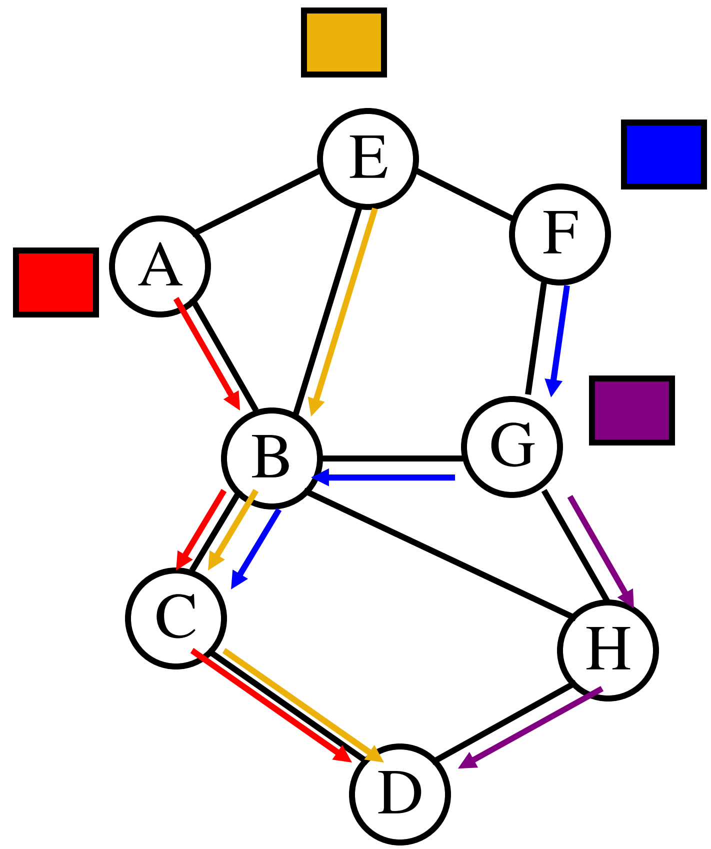
- Rich history [Wal88, Ber92]
  - Single queue, multiple parallel queues very well-understood
- Networks of queues
  - Hard to analyze owing to dependencies that arise downstream, even for independent packet arrivals
  - Kleinrock independence assumption
  - Fluid model abstractions
- Multiclass queuing networks:
  - Multiple classes of packets
  - Packet arrivals by time-invariant independent processes
  - Service times within a class are indistinguishable
  - Possible priorities among classes

Load Conditions & Stability

- Stability:
  - Finite upper bound on queues & delays
- Load constraint:
  - The rate at which packets need to traverse an edge should not exceed its capacity
- Load conditions are not sufficient to guarantee stability of a greedy queuing policy [LK91, RS92]
  - FIFO can be unstable for arbitrarily small load [Bra94]
  - Different service distributions for different classes
- For independent and time-invariant packet arrival distributions, with class-independent service times [DM95, RS92, Bra96]
  - FIFO is stable as long as basic load constraint holds

Adversarial Queuing Theory

- Directed network
- Packets, released at source, travel along specified paths, absorbed at destination
- In each step, at most one packet sent along each edge
- Adversary injects requests:
  - A request is a packet and a specified path
- Queuing policy decides which packet sent at each step along each edge
- [BKR+96, BKR+01]

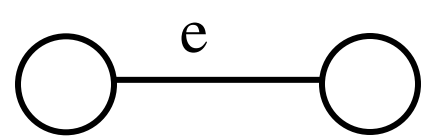


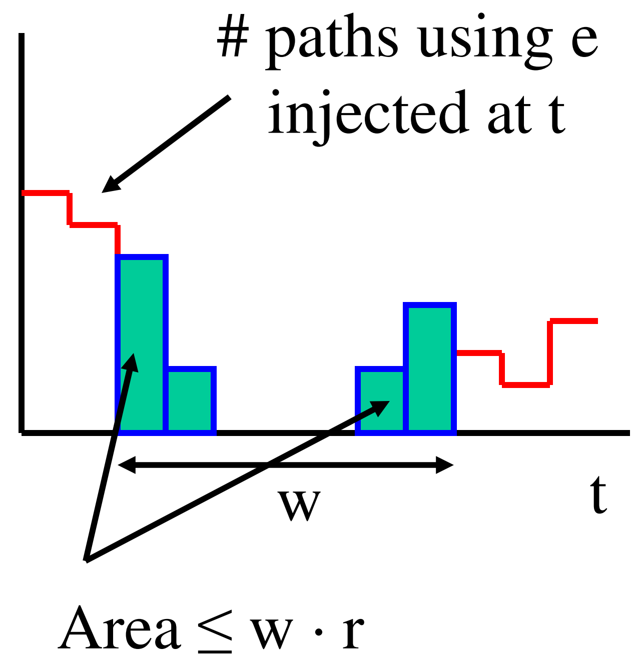
Load Constraints

- Let $N(T,e)$ be number of paths injected during interval T that traverse e
- (w,r)-adversary:
  - For any interval T of w consecutive time steps, for every edge e:

    $$N(T,e) \le w \cdot r$$
  - Rate of adversary is r
- (w,r) stochastic adversary:
  - For any interval [t+1…t+w], for every edge e:

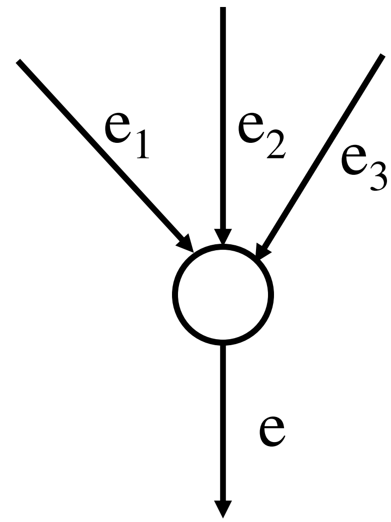
    $$E[N(T,e)|H_t] \le w \cdot r$$





Stability in DAGs

- Theorem: For any dag, any greedy policy is stable against any rate-1 adversary
- $A_t(e)$ = # packets in network at time t that will eventually use e
- $Q_t(e)$ = queue size for e at time t
- Proof: time-invariant upper bound on $A_t(e)$



Large queue: $Q_{t-w}(e) \geq w \Rightarrow A_t(e) \leq A_{t-w}(e)$

Small queue: $Q_{t-w}(e) < w \Rightarrow A_{t-w}(e) \leq w + \sum_j A_{t-w}(e_j)$

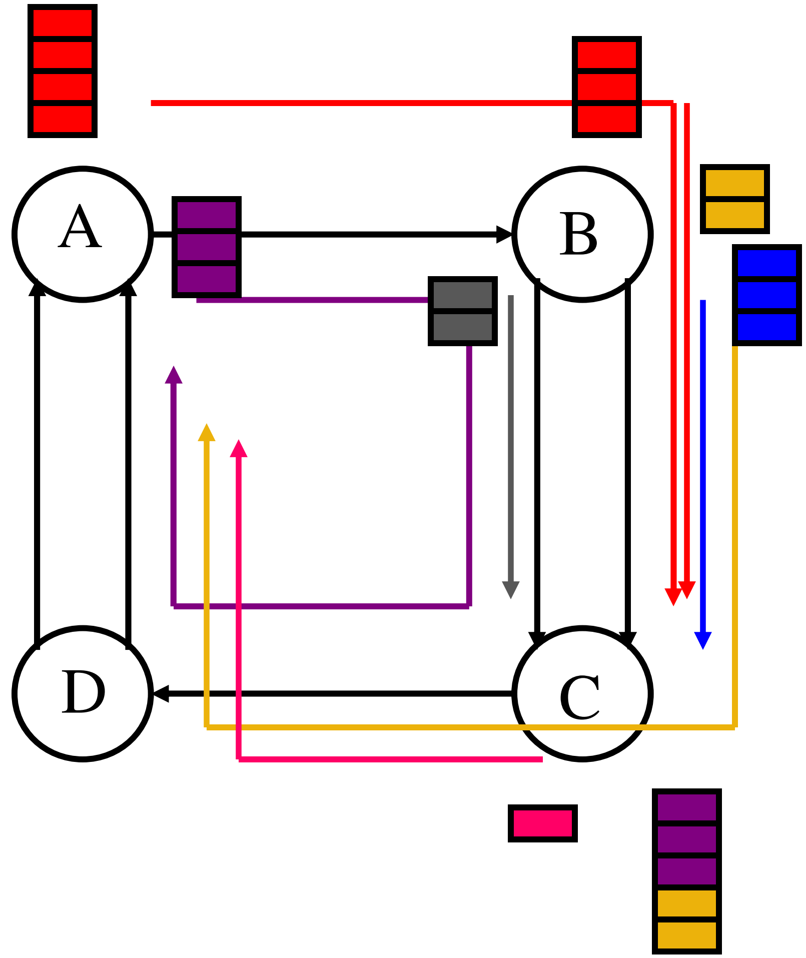
$$A_t(e) \leq 2w + \sum_j A_{t-w}(e_j)$$

Extension to Stochastic Adversaries

- Theorem: In DAGs, any greedy policy is stable against any stochastic 1-ε rate adversary, for any $\varepsilon>0$
- Cannot claim a hard upper bound on $A_t(e)$
- Define a potential φ_t, that is an upper bound on the number of packets in system
- Show that if the potential is larger than a specified constant, then there is an expected decrease in the next step
- Invoke results from martingale theory to argue that $E[\varphi_t]$ is bounded by a constant

FIFO is Unstable [A+ 96]

- Initially: s packets waiting at A to go to C
- Next s steps:
  - rs packets for loop
  - rs packets for B-C
- Next rs steps:
  - r^2s packets from B to A
  - r^2s packets for B-C
- Next r^2s steps:
  - r^3s packets for C-A
- Now: s+1 packets waiting at C going to A
- FIFO does not use edges most effectively



Stability in General Networks

- LIS and SIS are universally stable against rate <1 adversaries [AAF+96]
- Furthest-To-Go and Nearest-To-Origin are stable even against rate 1 adversaries [Gam99]
- Bounds on queue size:
  - Mostly exponential in the length of the shortest path
  - For DAGs, Longest-In-System (LIS) has poly-sized queues
- Bounds on packet delays:
  - A variant of LIS has poly-sized packet delays

Exercise

- Are the following two equivalent? Is one stronger than the other?
  - A finite bound on queue sizes
  - A finite bound on delay of each packet

Queuing Theory: Summary

- Focus on input dynamics in static networks
- Both stochastic and adversarial models
- Primary concern: stability
  - Finite bound on queue sizes
  - Finite bound on packet delays
- Algorithmic techniques: simple greedy policies
- Analysis techniques:
  - Potential functions
  - Markov chains and Markov decision processes
  - Martingales

Network Evolution

How do Networks Evolve?

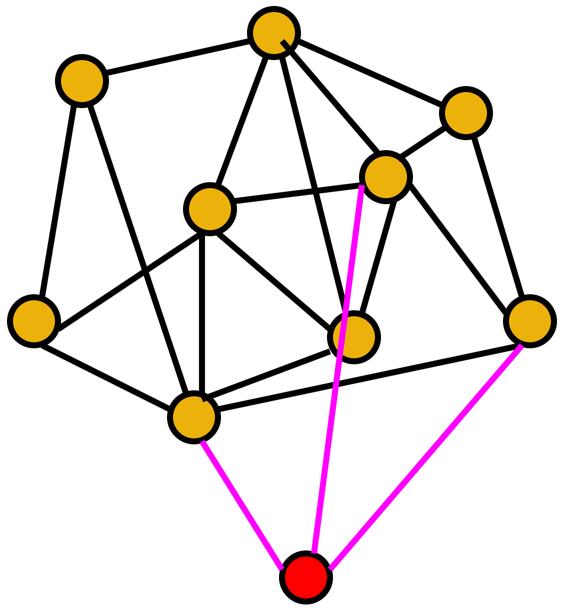
- Internet
  - New random graph models
  - Developed to support observed properties
- Peer-to-peer networks
  - Specific structures for connectivity properties
  - Chord [SMK+01], CAN [RFH+01], Oceanstore [KBC+00], D2B [FG03], [PRU01], [LNBK02], …
- Ad hoc networks
  - Connectivity & capacity [GK00…]
  - Mobility models [BMJ+98, YLN03, LNR04]

Internet Graph Models

- Internet measurements [FFF99, TGJ+02, …]:
  - Degrees follow heavy-tailed distribution at the AS and router levels
  - Frequency of nodes with degree d is proportional to $1/d^{\beta}$, $2 < \beta < 3$
- Models motivated by these observations
  - Preferential attachment model [BA99]
  - Power law graph model [ACL00]
  - Bicriteria optimization model [FKP02]

Preferential Attachment

- Evolutionary model [BA99]
- Initial graph is a clique of size d+1
  - d is degree-related parameter
- In step t, a new node arrives
- New node selects d neighbors
- Probability that node j is neighbor is proportional to its current degree
- Achieves power law degree distribution



Power Law Random Graphs

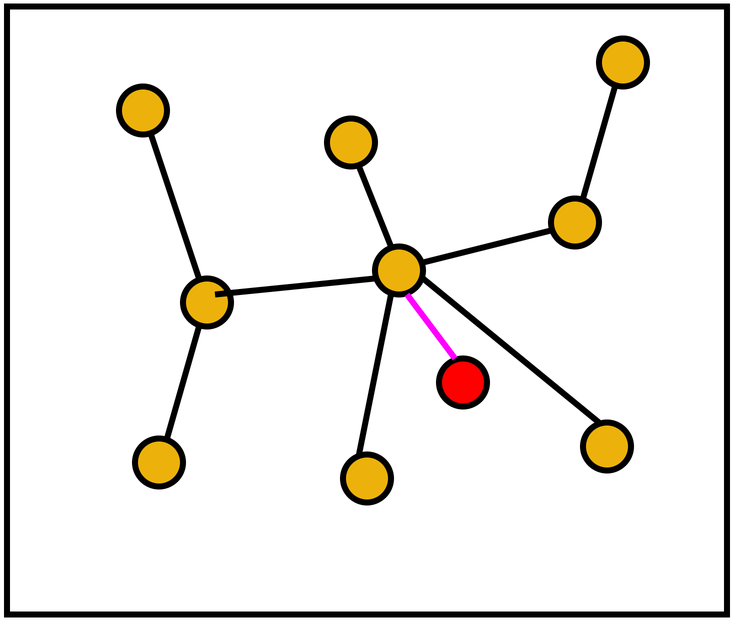
- Structural model [ACL00]
- Generate a graph with a specified degree sequence $(d_1,\ldots,d_n)$
  - Sampled from a power law degree distribution
- Construct d_j mini-vertices for each j
- Construct a random perfect matching
- Graph obtained by adding an edge for every edge between mini-vertices
- Adapting for Internet:
  - Prune 1- and 2-degree vertices repeatedly
  - Reattach them using random matchings

Bicriteria Optimization

- Evolutionary model
- Tree generation with power law degrees [FKP02]
- All nodes in unit square
- When node j arrives, it attaches to node k that minimizes:

$$\alpha \cdot d_{jk} + h_k$$

- If $4 \leq \alpha \leq o(\sqrt{n})$:
  - Degrees distributed as power law for some β, dependent on α
- Can be generalized, but no provable results known



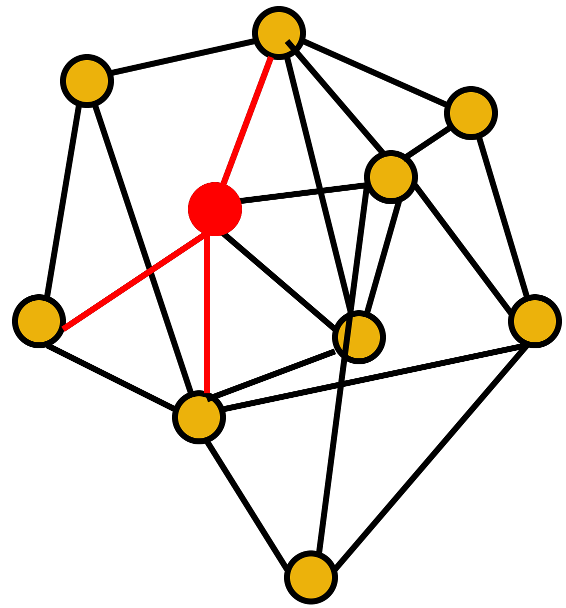
h_k: measure of centrality of k in tree

Connectivity & Capacity Properties

- Congestion in certain uniform multicommodity flow problems:
  - Suppose each pair of nodes is a source-destination pair for a unit flow
  - What will be the congestion on the most congested edge of the graph, assuming uniform capacities
  - Comparison with expander graphs, which would tend to have the least congestion
- For power law graphs with constant average degree, congestion is $O(n \log^2 n)$ with high probability [GMS03]
  - $\Omega(n)$ is a lower bound
- For preferential attachment model, congestion is $O(n \log n)$ with high probability [MPS03]
- Analysis by proving a lower bound on conductance, and hence expansion of the network

Network Creation Game

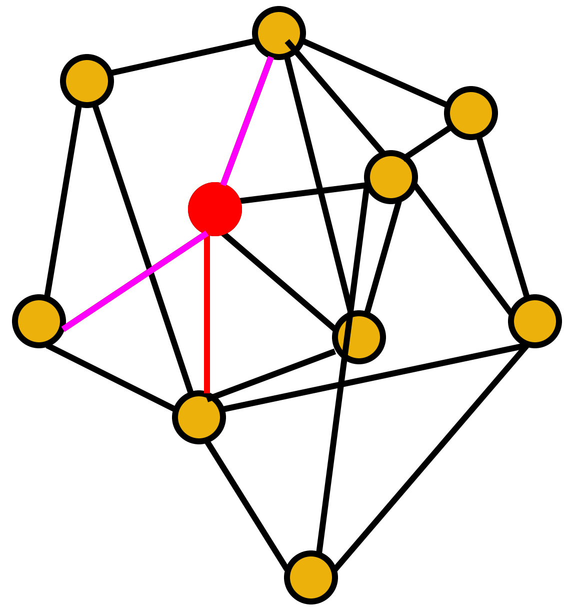
- View Internet as the product of the interaction of many economic agents
- Agents are nodes and their strategy choices create the network
- Strategy s_j of node j:
  - Edges to a subset of the nodes
- Cost c_j for node j:
  - $\alpha \cdot |s_j| + \Sigma_k \, d_{G(s)}(j,k)$
  - Hardware cost plus quality of service costs



3α + sum of distances to all nodes

Network Creation Game

- In the game, each node selects the best response to other nodes' strategies
- Nash equilibrium s:
  - For all j, $c_j(s) \le c_j(s')$ for all s' that differ from s only in the jth component
- Price of anarchy [KP99]:
  - Maximum, over all Nash equilibria, of the ratio of total cost in equilibrium to smallest total cost
- Bound, as a function of α [AEED06]:
  - $O(1)$ for $\alpha = O(\sqrt{n})$ or $\Omega(n \log n)$
  - Worst-case ratio $O(n^{1/3})$



Other Network Games

- Variants of network creation games
  - Weighted version [AEED06]
  - Cost and benefit tradeoff [BG00]
- Cost sharing in network design [JV01, ADK04, GST04]
- Congestion games [RT00, Rou02]
  - Each source-destination pair selects a path
  - Delay on edge is a function of the number of flows that use the edge

Network Evolution: Summary

- Models:
  - Stochastic
  - Game-theoretic
- Analysis techniques:
  - Graph properties, e.g., expansion, conductance
  - Probabilistic techniques
  - Techniques borrowed from random graphs